

# Regularised Supermembrane Theory and Static Configurations of M-Theory

(Static Matrix Model)

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## Abstract

We suggest that the static configurations of M-theory may be described by the matrix regularisation of the supermembrane theory in static regime. We compute the long range interaction between an M2-brane and an anti-M2-brane in agreement with the 11 dimensional supergravity result.

## 1 Introduction

The proposed M(atrix) model [1] as a non-perturbative formulation of M-Theory [2] has provided a new and effective framework for studying dualities and connections between different string theories [3, 4, 5, 6, 7]. This model is the dimensional reduction of 9+1  $U(N)$  SYM theory to 0+1 dimension [8] in large  $N$  limit, which latter was introduced and studied as the dynamics of  $N$  D0-branes [9, 10],

In the initial developments of the supermembrane theory [11, 12] (in the 11 dimensional supergravity background) it was observed that the existence of  $\kappa$ -symmetry imposes restrictions on the background fields which reduce to the 11 dimensional supergravity field equations. Since M-theory has the 11 dimensional supergravity as its low energy limit, the above observation suggests that every definition of M-theory should be in close connection to supermembrane theory. Thus, M-theory in infinite momentum frame and supermembrane action in light cone gauge written in a matrix form are related [1].

On the other hand the notion of a sub-structure in the formulation of the M(atrix) model for M-theory has played a central role. Therefore it is plausible to expect that the same sub-structure in the form of a matrix formulation should play a role in the framework describing the static configurations of the M-theory.

As there is no definition for covariant M-theory, it is tempting to study it in various gauges: light-cone, static, etc. . The above mentioned relations between supermembrane

theory and M-theory in light-cone gauge motivates us to search for the similar relation in static gauge. Our starting point is the action of supermembranes in 11 dimensions. By restricting the action to the static part of its phase space we obtain an action which after fixing its  $\kappa$ -symmetry can be written in a matrix form.

The resulted matrix action is invariant under  $SO(9)$  rotations of target space. Also the action has a gauge symmetry which corresponds to the world volume area preserving symmetry. Despite of the existence of the gauge symmetry, the interpretation of the model as a dimensional reduction of a SYM theory seems impossible.

We introduce solutions of the action, which as is expected from M-theory, have vanishing quantum corrections. Also we calculate the long range interaction of parallel M2-brane and anti-M2-brane solutions of the matrix model. The result is  $W(r) \sim \frac{1}{r^6}$  which agrees with the uncompactified 11 dimensional supergravity directly in contrast to the light-cone M(atrrix) theory result in compactified limit  $W(r) \sim \frac{1}{r^5}$ .

Conventions and some calculations are gathered in appendices.

## 2 Static supermembrane action as a matrix model

We use the following notations everywhere:

$$a, b = 0, 1, 2; \quad \mu, \nu = 0, 1, \dots, 9, 10; \quad I, J, K = 1, 2, \dots, 9, 10; \quad i, j, k = 1, 2, \dots, 9.$$

The supermembrane action in 11 dimensions is [13, 11]

$$S = \frac{-1}{2} \int d^3\eta \left( 2\sqrt{-g} + \epsilon^{abc} \bar{\theta} \Gamma_{\mu\nu} \partial_a \theta \times (\Pi_b^\mu \partial_c X^\nu + \frac{1}{3} \bar{\theta} \Gamma^\mu \partial_b \theta \bar{\theta} \Gamma^\nu \partial_c \theta) \right), \quad (1)$$

where  $\Pi$ 's and  $g$  are

$$\Pi_a^\mu = \partial_a X^\mu + \bar{\theta} \Gamma^\mu \partial_a \theta, \quad g_{ab} = \Pi_a \cdot \Pi_b, \quad (2)$$

and  $\theta$  is eleven dimensional Majorana spinor.

The action (1) is invariant under global SUSY transformation

$$\delta X^\mu = -\bar{\epsilon} \Gamma^\mu \theta, \quad \delta \theta = \epsilon, \quad (3)$$

and also under the local fermionic symmetry,  $\kappa$ -symmetry

$$\delta X^\mu = \bar{\kappa} (1 - \Gamma) \Gamma^\mu \theta, \quad \delta \theta = (1 - \Gamma) \kappa, \quad (4)$$

where

$$\Gamma = \frac{\epsilon^{abc}}{6\sqrt{-g}} \Pi_a^\mu \Pi_b^\nu \Pi_c^\rho \Gamma_{\mu\nu\rho}, \quad \Gamma^2 = 1.$$

We decompose the coordinates as  $\eta_a = (\tau, \sigma_r)$ ,  $r = 1, 2$ .

We go to the static regime defined by

$$X^0 \equiv \tau, \quad \dot{X}^I \equiv \dot{\theta} \equiv 0; \quad (5)$$

then the components of  $g$  are found to be

$$\begin{aligned} g_{00} &= -1, & -f_r &\equiv g_{0r} = -\bar{\theta}\Gamma^0\partial_r\theta, \\ g_{rs} &= \bar{g}_{rs} - f_r f_s, & \bar{g}_{rs} &\equiv \Pi_{rI}\Pi_{sI}; \end{aligned} \quad (6)$$

and it can easily be shown that,

$$g = -\bar{g}, \quad \bar{g} = \det \bar{g}_{rs} = \frac{1}{2} \epsilon^{rs} \epsilon^{r's'} \bar{g}_{rr'} \bar{g}_{ss'} = \frac{1}{2} (\epsilon^{rs} \Pi_r^I \Pi_s^J)^2. \quad (7)$$

Putting all the above relations in (1), we obtain

$$S = \frac{1}{2} \int d\tau d^2\sigma \left( -e^{-1} - e \bar{g} - 2\epsilon^{rs} \bar{\theta} \Gamma_{0I} \partial_r \theta \partial_s X^I - \epsilon^{rs} \bar{\theta} \Gamma_{0I} \partial_r \theta \bar{\theta} \Gamma^I \partial_s \theta \right), \quad (8)$$

where  $e$  appears as an auxiliary field for linearising the action; its equation of motion gives

$$e^2 \bar{g} = 1, \quad (9)$$

which can be used for eliminating  $e$ . Due to (9), configurations with  $\bar{g} = 0$  are unacceptable.

The action (1) has a local fermionic symmetry, called  $\kappa$ -symmetry which allows one to gauge away half of the fermionic degrees of freedom of  $\theta$ .  $\theta$  is a 32-component 11-dimensional Majorana spinor and is real in a real representation of  $\Gamma$  matrices which we use (see appendix). We fix the  $\kappa$ -symmetry just as by the light cone gauge <sup>1</sup> (i.e.  $(\Gamma^0 + \Gamma^{10})\theta = \Gamma^+\theta = 0$ )

$$\theta = \frac{1}{2} \begin{pmatrix} \lambda \\ \lambda \end{pmatrix}, \quad \lambda = \lambda^*; \quad (10)$$

then it can be shown that

$$\begin{aligned} \bar{\theta} \Gamma_i \partial \theta &= 0, & \bar{\theta} \Gamma_{10} \partial \theta &= -\frac{1}{2} \lambda^T \partial \lambda, \\ \bar{\theta} \Gamma_{0i} \partial \theta &= -\frac{1}{2} \lambda^T \gamma_i \partial \lambda, & \bar{\theta} \Gamma_{0,10} \partial \theta &= 0. \end{aligned} \quad (11)$$

After integration over  $\tau$  ( which gives  $\mathcal{T}$ ) the action (8) takes the following form

$$S = -\frac{1}{2} \mathcal{T} \int d^2\sigma e^{-1} \left( \frac{1}{2} \{X^i, X^j\}^2 + (\{X^i, X^{10}\} - \frac{1}{2} \lambda^T \{X^i, \lambda\})^2 + \lambda^T \gamma_i \{X^i, \lambda\} + 1 \right), \quad (12)$$

where

$$\{a, b\} = e (\partial_{\sigma_1} a \partial_{\sigma_2} b - \partial_{\sigma_2} a \partial_{\sigma_1} b) = e \epsilon^{rs} \partial_r a \partial_s b, \quad (13)$$

which satisfies the Jacobi identity.

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<sup>1</sup>In fact we could do gauge fixing before restricting the action to its static regime by the ansatz (5).

We can now formulate our matrix model. By usual substitutions[13, 1, 14] <sup>2</sup>

$$\{a, b\} \Rightarrow -i [a, b], \quad \int e^{-1} d^2 \sigma \Rightarrow Tr, \quad (14)$$

with the following consequences

$$\begin{aligned} \int e^{-1} d^2 \sigma \left( \{a, b\} c \right) &= \int e^{-1} d^2 \sigma \left( a \{b, c\} \right) \Rightarrow Tr \left( [a, b] c \right) = Tr \left( a [b, c] \right), \\ \int e^{-1} d^2 \sigma \{a, b\} &= 0 \Rightarrow Tr[a, b] = 0, \end{aligned} \quad (15)$$

one finds

$$\begin{aligned} S = & - \frac{1}{2} \alpha \mathcal{T} Tr \left( \frac{1}{2} [X^i, X^j]^2 + ([X^i, X^{10}] - \gamma \frac{1}{2} \lambda^T [X^i, \lambda])^2 + i \lambda^T \gamma_i [X^i, \lambda] \right) \\ & + \frac{1}{2} \beta \mathcal{T} Tr (1). \end{aligned} \quad (16)$$

Here  $\alpha$ ,  $\beta$  and  $\gamma$  appeared due to dimensional considerations in going from the bracket to the commutator and also from integration to trace. We will fix  $\alpha$  and  $\beta$  later.

The action (16) has a gauge symmetry which may be identified with area-preserving symmetry of the supermembrane [13]. It is defined by an arbitrary matrix  $\Lambda$

$$\begin{aligned} \delta_{gauge} X^i &= i[X^i, \Lambda], \\ \delta_{gauge} \lambda &= i[\lambda, \Lambda], \\ \delta_{gauge} X^{10} &= i[X^{10}, \Lambda]. \end{aligned} \quad (17)$$

Furthermore the action (16) is invariant under SUSY transformations

$$\begin{aligned} \delta X^i &= 0, \\ \delta X^{10} &= \frac{1}{2} \eta^T \lambda, \\ \delta \lambda &= \eta, \end{aligned} \quad (18)$$

with  $\eta$  as an anti-commuting  $SO(9)$  spinor and it can be shown that the above transformations form space-time SUSY algebra

$$\begin{aligned} [\delta_\eta, \delta_{\eta'}] X^i &= 0, \\ [\delta_\eta, \delta_{\eta'}] X^{10} &= \eta'^T \eta, \\ [\delta_\eta, \delta_{\eta'}] \lambda &= 0, \end{aligned} \quad (19)$$

which for  $X^{10}$  can be understood as a non-zero translation, due to  $\{q_A, q_B\} = \Gamma^{10} P_{10} \delta_{AB}$ . Here 10-th direction is appearing as the 11-th direction in the super-Galilean algebra [1, 15] <sup>3</sup>.

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<sup>2</sup>There is a factor  $n$  for  $n \times n$  matrices in going from bracket to commutator and also from integration to trace. Here we absorbed the factor every time in commutator entries.

<sup>3</sup>In general to find the complete SUSY transformations, one must search those which respect  $\kappa$ -

### 3 Solutions with vanishing quantum corrections

In this section we describe certain configurations which are the solutions of the classical equations of motion, and it will be shown that the quantum corrections at one-loop order vanish for them. So these solutions, as is expected from similar ones in M-theory, show BPS behaviour.

The one-loop effective action around the classical solutions

$$X^{10} = \lambda = 0,$$

is computed in the appendix and the result is

$$W = \frac{1}{2} Trlog\left(P_i^2 \delta_{IJ} - 2i F_{ij}\right) - \frac{1}{4} Trlog\left(P_i^2 + \frac{i}{2} F_{ij} \gamma^{ij}\right) - Trlog(P_i^2), \quad (20)$$

with the following definitions

$$P_i * = [p_i, *], \quad F_{ij} * = [f_{ij}, *], \quad f_{ij} = i[p_i, p_j], \quad (21)$$

where  $p_i$  is classical solution of  $X_i$ .

Every solution with

$$F_{ij} = 0, \quad \forall i, j, \quad (22)$$

leads to vanishing of the one-loop effective action, due to the following algebra

$$W \sim \left(\frac{10}{2} - \frac{16}{4} - 1\right) Trlog(P_i^2) = 0.$$

In the following we search for these solutions <sup>4</sup>.

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symmetry gauge fixing solving the equation

$$\Gamma^+ \theta = 0 \leftrightarrow \Gamma^+ (\theta + \epsilon + (1 - \Gamma) \kappa) = 0 \Rightarrow \Gamma^+ (\epsilon + (1 - \Gamma) \kappa) = 0.$$

This is a constraint equation between SUSY and  $\kappa$ -symmetry parameters  $\epsilon$  and  $\kappa$  as a global and local spinors respectively. A rapid solution is  $\kappa = 0$  and  $\epsilon \sim \begin{pmatrix} \eta \\ \eta \end{pmatrix}$ , which leads to SUSY transformations (18). Another closed form solutions seemed inaccessible in our static case. A similar observation is reported as a result of non-linearities of equations of motion [12]. So we just keep (18).

<sup>4</sup>The point-like configurations which may be represented by the following solutions

$$X^i = diag(x_1^i, x_2^i, \dots, x_n^i), \quad X^{10} = \lambda = 0,$$

are not acceptable because of vanishing  $\bar{g}$  in (9). It is in agreement with the fact that the individual 11 dimensional supergravitons which are candidates for "quark" substructure of our model (due to their role in infinite momentum frame M(atr)ix model as "partons") can not be studied as static configurations in 11 dimensions, because they are massless. This argument also will be supported by the equation of motion of  $n$ , the size of matrices. By inserting solutions introduced above, in the action one finds,

$$S = 0 + \frac{1}{2} \beta T n.$$

The equation of motion for  $n$  has no solution (gives  $1 = 0$ ).

To begin with we consider a solution of (12) which represents a single flat static membrane. With the conditions  $X^{10} = \lambda = 0$ , the equations of motion (12) are

$$\{X^i, \{X^i, X^j\}\} = 0.$$

Then

$$X^1 = \sigma_1, \quad X^2 = \sigma_2, \quad \text{other } X^i{}'_s = 0, \quad (23)$$

represent a single membrane solution,  $\{X^1, X^2\} = \{\sigma_1, \sigma_2\} = e$  ( $=1$ , due to the equation of motion of  $e$ ). In the matrix version the conditions  $X^{10} = \lambda = 0$  give

$$[X^i, [X^i, X^j]] = 0,$$

which in analogy with (23) leads to

$$X^1 = \frac{L_1}{\sqrt{2\pi n}}q, \quad X^2 = \frac{L_2}{\sqrt{2\pi n}}p, \quad \text{other } X^i{}'_s =, \quad (24)$$

with  $[q, p] = i$  and  $0 \leq q, p \leq \sqrt{2\pi n}$  eigenvalue distributions. This solution represents a 2 dimensional object extended in  $X^1$  and  $X^2$  directions, and clearly it satisfies (22) and so is stable under quantum fluctuations. Also due to the spectrum of  $p$  and  $q$  the area of the 2 dimensional object (M2-brane) is  $L_1 L_2$ .

There are also solutions corresponding to two parallel M-2-branes,

$$\begin{aligned} X^1 &= \begin{pmatrix} \frac{L_1}{\sqrt{2\pi n}}q & 0 \\ 0 & \frac{L_1}{\sqrt{2\pi n}}q \end{pmatrix} \equiv p^1, \quad X^2 = \begin{pmatrix} \frac{L_2}{\sqrt{2\pi n}}p & 0 \\ 0 & \frac{L_2}{\sqrt{2\pi n}}p \end{pmatrix} \equiv p^2, \\ X^3 &= \begin{pmatrix} r/2 & 0 \\ 0 & -r/2 \end{pmatrix} \equiv p^3, \quad \text{other } X^i{}'_s = 0, \end{aligned} \quad (25)$$

extending in  $X^1$  and  $X^2$  directions and at the distance  $r$  in  $X^3$  direction. Again clearly this solution satisfies (22) which means that the two M2-branes are under no-force condition.

## 4 M2-brane and anti-M2-brane long range interaction

In this section we calculate the long range interaction between two parallel M2-brane and anti-M2-brane. Solutions corresponding to two membranes with opposite charges were introduced in [16]

$$\begin{aligned} X^1 &= \begin{pmatrix} \frac{L_1}{\sqrt{2\pi n}}q & 0 \\ 0 & \frac{L_1}{\sqrt{2\pi n}}q \end{pmatrix} \equiv p^1, \quad X^2 = \begin{pmatrix} \frac{L_2}{\sqrt{2\pi n}}p & 0 \\ 0 & -\frac{L_2}{\sqrt{2\pi n}}p \end{pmatrix} \equiv p^2, \\ X^3 &= \begin{pmatrix} r/2 & 0 \\ 0 & -r/2 \end{pmatrix} \equiv p^3, \quad \text{other } X^i{}'_s = X^{10} = \lambda = 0, \end{aligned} \quad (26)$$

with,  $[q, p] = i$ . To calculate the potential between these membranes one must find the one-loop effective action of (16). The one-loop effective action  $W$  was introduced in the previous section (and calculated in the appendix)

$$W = \frac{1}{2} \text{Tr} \log \left( P_i^2 \delta_{IJ} - 2i F_{ij} \right) - \frac{1}{4} \text{Tr} \log \left( P_i^2 + \frac{i}{2} F_{ij} \gamma^{ij} \right) - \text{Tr} \log (P_i^2), \quad (27)$$

with  $P_i * = [p_i, *]$ ,  $F_{ij} * = [f_{ij}, *]$ ,  $f_{ij} = i[p_i, p_j]$ .

The calculation of (27) with solutions like (26) are similar to those of [14] for calculating the interaction between two anti-parallel D-strings. For solutions (26) we have  $[p_i, f_{ij}] = c - \text{number}$  which means that  $P_i^2$  and  $F_{ij}$  are simultaneously diagonalisable. Also  $[P_1, P_2] \sim i$ , which means that  $P_i^2$  behaves like a harmonic oscillator. The steps of calculations are presented in [14] and the result is as follows

$$W = (-8n) \left( \frac{L_1 L_2}{2\pi n} \right)^3 \frac{1}{r^6} + O\left(\frac{1}{r^8}\right), \quad (28)$$

which is in agreement with 11 dimensional supergravity results for interaction of M2-brane and anti-M2-brane [16, 17]. It is notable that this result is in the uncompactified limit of 11 dimensional supergravity, in contrast to the result of light-cone M(atr)ix theory ( $W(r) \sim \frac{1}{r^5}$ ) [16].

The result (28) can be used for fixing the parameters  $\alpha$  and  $\beta$  in (16). By inserting (24) in (16) one finds

$$S = \left(\frac{1}{4}\right) \alpha \mathcal{T} \left(\frac{L_1 L_2}{2\pi n}\right)^2 n + \frac{\beta \mathcal{T}}{2} n, \quad (29)$$

and the equation of motion of  $n$  gives

$$\frac{L_1 L_2}{2\pi n} = \sqrt{\frac{2\beta}{\alpha}}, \quad (30)$$

resulting in

$$S = \frac{1}{2\pi} \sqrt{\frac{\alpha\beta}{2}} (\mathcal{T} L_1 L_2) = T_M (\mathcal{T} L_1 L_2). \quad (31)$$

which the second equality is the action of a flat membrane with  $T_M$  as its tension. (31) gives

$$T_M = \frac{1}{2\pi} \sqrt{\frac{\alpha\beta}{2}}. \quad (32)$$

Also by comparing (28) with 11 dimensional supergravity interaction [16] one finds

$$\frac{L_1 L_2}{2\pi n} = \sqrt{\frac{24\pi \mathcal{T}}{T_M}}. \quad (33)$$

By using (30,32,33) and extracting an irrelevant numerical factor,  $\alpha$  and  $\beta$  are fixed as follows

$$\alpha = \sqrt{\frac{T_M^3}{\mathcal{T}}}, \quad \beta = 12\pi\sqrt{T_M\mathcal{T}}. \quad (34)$$

By choosing  $\mathcal{T} = T_M^{-1/3}$  the action (16) becomes

$$\begin{aligned} S = & - \frac{1}{2} T_M^{4/3} \text{Tr} \left( \frac{1}{2} [X^i, X^j]^2 + ([X^i, X^{10}] - \gamma \frac{1}{2} \lambda^T [X^i, \lambda])^2 + i \lambda^T \gamma_i [X^i, \lambda] \right) \\ & + 6\pi \text{Tr} (1). \end{aligned} \quad (35)$$

## 5 Conclusion and discussions

In this letter we introduced a matrix model of static configurations of M-theory. By construction the large  $n$ -limit of the model, at least classically, is equivalent with static supermembrane action after  $\kappa$ -symmetry gauge fixing. We calculated the long range interaction of an M2-brane and an anti-M2-brane solution in this model and found to be in agreement with the 11 dimensional supergravity results.

M-theory is supposed to reduce to various string theories and their compactifications. However a model for static configurations of M-theory can not be interpreted exactly as a string theory, because there are static configuration in string theories which are not static in uncompactified M-theory (e.g. non-moving D0-branes in IIA theory which are known as KK modes of massless supergravitons of 11 dimensional supergravity, and so they move with speed of light in 11 dimensions.). Notice that the reverse of the above argument is not valid, i.e. static configurations in M-theory remain static after compactification. So compactifications of the static matrix model is specially interesting.

### Appendix 1- Conventions and notations

Signatures:  $g_{ab} = (-, +, +)$ ,  $\eta_{\mu\nu} = (-, +, +, +, +, +, +, +, +, +, +)$ ,  
 $\epsilon^{0rs} = -\epsilon^{rs}$ ,  $\bar{\theta} = \theta^\dagger \Gamma_0$ ,  $[\Gamma^\mu, \Gamma^\nu]_+ = 2\eta^{\mu\nu}$ ,  $\Gamma^{\mu\dagger} = \Gamma^0 \Gamma^\mu \Gamma^0$ ,  $\Gamma^{\mu\nu} = 1/2 (\Gamma^\mu \Gamma^\nu - \Gamma^\nu \Gamma^\mu)$ ,  
 $\Gamma^0 = \begin{pmatrix} 0 & -1_{16} \\ 1_{16} & 0 \end{pmatrix}$ ,  $\Gamma^{10} = \begin{pmatrix} 1_{16} & 0 \\ 0 & -1_{16} \end{pmatrix}$ ,  $\Gamma^i = \begin{pmatrix} 0 & \gamma_{16}^i \\ \gamma_{16}^i & 0 \end{pmatrix}$ ,  
 $\Gamma^+ = \Gamma^0 + \Gamma^{10}$ ,  $\gamma_{16}^{i\dagger} = \gamma_{16}^{i*} = \gamma_{16}^i$ ,  $[\gamma^i, \gamma^j]_+ = 2\delta^{ij}$ ,  $\Gamma^1 \Gamma^2 \dots \Gamma^9 \Gamma^{10} = \Gamma^0$ .

### Appendix 2- One-loop effective action

The calculation of this part is similar to those of [14]. In this part we decompose the matrices  $X$ 's and  $\theta$ 's to classical solutions and quantum fluctuations as follows,

$$\begin{aligned} X^i &= (p^i)_{class.} + a^i, \\ \lambda &= (0)_{class.} + \phi, \\ X^{10} &= (0)_{class.} + a^{10}, \end{aligned} \quad (36)$$

where  $(\dots)_{class.}$  are classical solutions and the remainder of RHS's are quantum fluctuations around classical solutions. After expanding the action (16) up to quadratic terms in fluctuations and using equations of motion one finds

$$\Delta S = -\text{Tr} \left( \frac{1}{2} [p_i, a_J]^2 + [p_i, p_j][a_i, a_j] - \frac{1}{2} [p_i, a_i]^2 + \frac{i}{2} \phi^T \gamma^i [p_i, \phi] \right). \quad (37)$$



We have ghosts, because of the gauge invariance introduced in the text,

$$S_{ghost} = -Tr\left(\frac{1}{2}[p_i, a_i]^2 + [p_i, b][p_i, c]\right).$$

By introducing the adjoint operators

$$P_i * = [p_i, *], \quad F_{ij} * = [f_{ij}, *], \quad f_{ij} = i[p_i, p_j], \quad (38)$$

the final form of the action will be as follows

$$S_2 = Tr\left(\frac{1}{2}(a_I P_i^2 \delta_{IJ} a_J - a_i 2i F_{ij} a_j) - \frac{i}{2} \phi^T \gamma^i P_i \phi + b P_i^2 c\right).$$

By inserting  $S_2$  in the path integral the one-loop effective action is obtained

$$\begin{aligned} W &= -\log \int [da][d\phi][dc][db] e^{-S_2} \\ &= \frac{1}{2} Tr \log \left( P_i^2 \delta_{IJ} - 2i F_{ij} \right) - \frac{1}{4} Tr \log \left( P_i^2 + \frac{i}{2} F_{ij} \gamma^{ij} \right) - Tr \log (P_i^2). \end{aligned} \quad (39)$$

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